



Hermann Graßmann

From Past to Future: Graßmann's Work in Context
Graßmann Bicentennial Conference, September 2009

Hans-Joachim Petsche
Albert C. Lewis
Jörg Liesen
Steve Russ
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Hans-Joachim Petsche
 Universität Potsdam
 Institut für Philosophie
 Am Neuen Palais 10
 14469 Potsdam
 Germany
petsche@uni-potsdam.de

Jörg Liesen
 TU Berlin
 Institut für Mathematik
 Straße des 17. Juni 136
 10623 Berlin
 Germany
liesen@math.tu-berlin.de

Albert C. Lewis
 Educational Advancement Foundation
 Rio Grande St. 2303
 Austin, TX 78705
 USA
alewis@edu-adv-foundation.org

Steve Russ
 University of Warwick
 Dept. Computer Science
 Coventry CV4 7AL
 United Kingdom
sbr@dcs.warwick.ac.uk

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Cover figure: Hermann Günther Graßmann, xylograph after a photograph from 1874 (mirror inverted). Source: Hermann Graßmann. *Gesammelte mathematische und physikalische Werke*. Bd. 1.1. Herausgeg. von Fr. Engel unter Mitwirkung von E. Study, Leipzig 1894; background: The New Palace campus of the University of Potsdam (Communs of the New Palace at Park Sanssouci, a UNESCO World Heritage Site).

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Illustration 2. Opening session of the Grassmann Bicentennial Conference in Potsdam



Illustration 3. Hermann Grassmann's great-grandson Dietrich Grassmann and the main organizer of the conference Hans-Joachim Petsche in front of a picture showing the Stettin Masonic lodge garden in the nineteenth century

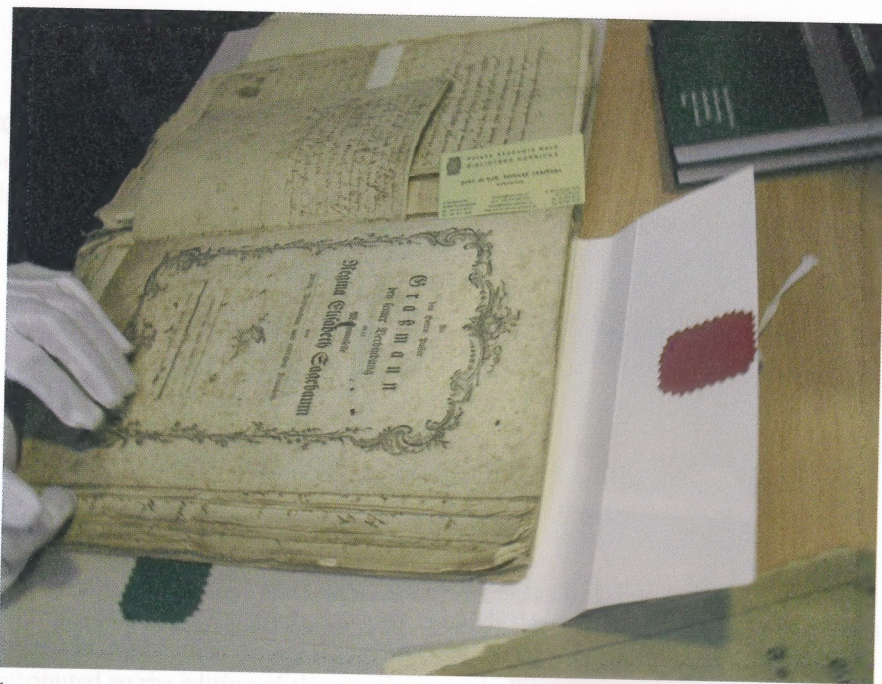


Illustration 4. Inauguration of a Grassmann exhibition unveiling recently discovered documents in the Pomeranian Library (during the conference's stay in Szczecin)



Illustration 5. Participants of the Grassmann Conference in front of the former Marienstiftsgymnasium in Szczecin

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The Rigveda Dictionary from a modern viewpoint

Toshifumi Gotō

The *Dictionary of the Rigveda* by Hermann Grassmann is the only standard Rigveda [RV] dictionary today.¹ The 5th reprint followed in 1976, and there exists a 6th edition, revised and supplemented by M. Kozianka [H. Grassmann 1996]. After its appearance, research has developed radically in the fields of philology (understanding of vocabulary and meaning) and grammar. The progress achieved in Indo-European linguistics in the twentieth century has been remarkable. The study of morphology including the functional analysis of grammatical forms has developed towards totally new dimensions. The *Dictionary*, nevertheless, is an essential tool even today. Its layout plays an important role in this regard. It seems that Grassmann could anticipate how the gaps should be bridged in the future, and how an entry would be more or less mechanically reinterpreted. I would like to report how this phenomenon was possible, foremost from my own three-year experience with the new translation of the Rigveda [Witzel and Gotō 2007], after the translations of Grassmann [RV1; RV2], [Ludwig 1876–1888], and [Geldner 1951] (completed in the 1920s, but published in 1951).

¹ Leipzig: F. A. Brockhaus 1873. Preface: Stettin, 18.8.1872. Afterword: 8.8.1875. For more information on the publication, cf. Preface, p. V and [Windisch 1920, 365].

Lemmas, forms and meaning

The *R̥gveda* is a collection of more than 10,000 *śloka*-s “praising stanzas”, codified as a sacred text (*veda*) presumably about 1,200 B.C. It is not only the oldest text in India, but also one of the most important documents written in the Indo-European languages. Grassmann’s dictionary is an indispensable tool for RV scholars and for Indo-European studies in general.

Grassmann records all forms in the RV with almost all the occurrences and essential contexts. The occurrence is cited, however, inconveniently according to 2-figure numbers (sequential hymn and stanza), and not by the 3-figure numbers (book, hymn, and stanza) which are common today. The 1st edition of the RV by [Aufrecht 1861; 1863], which Grassmann used as a basis, gives us also the continuous numbering besides the 3-figure system. Unfortunately, in today’s standard edition [Aufrecht 1877] the collateral data has been left out. The songs have been classified and codified according to the poets’ families; within a family book, the number of the songs and stanzas as well as the length of the metrical line was standard for the arrangement of the songs. All these facts became obvious in full detail only after Grassmann’s *Dictionary* had appeared, through investigations by [Bergaigne 1886; 1887] and [Oldenberg 1888]. But Grassmann had already essentially recognized this arrangement. He writes about this editorial principle in his translation of the RV [RV1, 2]. His translation consists of two parts, reflecting his understanding of the editorial background as well: 1. “The family books of the R̥g-Veda”: II–VIII [RV1], 2. “Collected books”: I, IX, X [RV2]. The information showing in which of the ten books a certain word form is attested is important. It is true that the 3-figure parameter causes substantially more difficulty for presentation and correction. Did the mathematician Grassmann confuse the philologist Grassmann here? Anyway, one cannot easily understand his decision. Maybe he had noticed the importance of this fact only gradually, during his translation, and did not have enough time for the rearrangement.

At first, let a group of verbal roots be taken up to illustrate Grassmann’s treatment, and to compare them with today’s state of research (“xxx” suggests omission; “583,10 [VII 67,10]” means: 583,10 in the *Dictionary* corresponds to Book 7, 67th Hymn, Stanza 10; this reference is omitted in Book I; [] is my addition):
column 479

³*jar*, originally **gvar*, substantially identical with *jur*, perhaps also with *jvar* [“be in fever”]. Basic meaning: “grind down, wear out, consume” (cf. Kuhn, in his *Zeitschrift für Vergleichende Sprachforschung*..., Curtius, *Grundzüge der griechischen Etymologie*,² 1866). (1) “grind down, make fragile, make old”; (2) “let become old”, i.e. “let reach an old age”; (3) “become fragile, become old, be consumed”; (4) “be ground down”; (5) causative “grind out, wear out, make old”.

stem *jára-*: -*anti* (1) *ná yám ~ śarādas* 465,7 [VI 24,7]. -*atam* (2nd person dual imperative) (2) *sūrīn* 583,10 [VII 67,10].

jara-: -*anta* (3) *pūrvīś uśāsas* 857,7 [X 31,7].

orist *jāriṣ-*: -*śus* (3) *mā dyumnāni* 139,8.

stem of causative *jarāya-*, *jārāya-* see under participle.

participle *jārat-* ((3) fragile, old, aged): -*antam* (3) xxx. -*antā* (dual) (3) xxx. -*atos* (genitive dual) (3) xxx. -*atas* (genitive singular) (3) xxx. -*atibhis* (4) xxx.

participle of causative *jarāyat-*: -*an* (5) xxx. -*antam* (5) xxx. -*antī* (5) xxx. -*antīs* (accusative plural) xxx.

participle of causative *jārāyat-* (Padapāṭha *jarāyat-*): -*antī* 124,10 (*uśās*).

column 493

jur, another form of ³*jar*. (1) “become fragile, become old”, of living beings, further (2) of the god’s deeds; (3) “make frail, make old”; especially (4) participle II *jūrṇā-* “friable, brittle” (of a net 180,5), “frail” (of the casted-off snake’s skin 798,44 [IX 86,44]); (5) *jūrṇā-* “very old”, without nuance of fragility. – Cf. *jūrv* und *ni-jūr*.

stem I. *jurá-*: -*átam* (2nd dual imperative) *pañés ásum* 182,3.

stem II. *jūrya-*: -*ati* (1) *sá (agnīs) ayā kṛpā ná ~* 128,2. -*anti* (2) *ná vām ~ pūrvīā kṛtāni* 117,4.

imperfect *ajūrya-*: -*an* (1) *devanīdas ha prathamās ~* 152,2.

participle *jurát-* (stem I): -*até* (1) *cyāvānāya* 584,6 [VII 68,6]. -*atām* (1) 225,10 [II 34,10] (*jārāya*).

jūryat- (stem II) (see *á-jūryat*): -*atsu* (1) *vāneṣu* (metaphorical) 257,1 [III 23,1], opposed to *ajáras (agnīs)*. -*antai* (dative feminine) (1) *pátim ~ adatta* 117,7.

participle perfect *jujurvás-*, weak *jujurús-*: (1) xxx

participle II *jūrṇā-*: xxx (4) xxx. xxx (5) xxx. xxx (4) xxx. xxx (5) xxx.

verbal *jūr-*, “becoming old, aged”, included in *a-*, *rta-*, *dhiyā-*, *sanā-jūr-*.

column 499

jūrv, basic meaning presumably “wear out, consume”, and therefore related to *jur*, and came from this (cf. *nijur-*); but limited in consuming by heat (of fire, lightening, sun), thus: “scorch, consume through the heat (of the sun)”.

With *nī*, “burn down, consume through the heat (of the fire, lightening)”. *sám*, “burn, consume through the heat (of the fire)”.

stem *jūrva-*: -*asi sám* 669,7 [VIII 60,7] *atasám*. -*ati nī* 1013,3 [X 187,3] *rākṣāmsi*. -*athas nī*: *rākṣas* 620,4 [VII 104,4]. -*ās nī*: *śátrum* 221,5 [II 30,5].

stem *jūrva-*: -*a nī*: *spṛdhas* 447,6 [VI 6,6].

orist *jūrvī-*: -*it nī*: *ámānuṣam* 202,10 [II 11,10].

participle *jūrvat-*: -*an* (read *jūruan*) *viśvāni* 191,9. – *nī* 303,11 [IV 7,11].

Note: The more basic form *jūr* is found in *ni-jūr* “scorching, consuming”.

1. Analysis of the entry

Here we encounter various appearances of the one Proto-Indo-European [PIE] verbal root $*\acute{g}erh_2$ “make old, infirm”. Its strong form (accented full-grade) is $*\acute{g}erh_2$ which is developed in Old Indo-Āryan [OIA] to $jár^i$, for example in the present stem $jár^i-a-ti$ “make old, make achieve old age”. The superscript i is only a sign for a reflex of one of three PIE laryngeals for linguists’ convenience; it appears as i in some circumstances, for example between consonants. “- ti ” is the ending of the 3rd singular active as representation. Greek γέρων (*gérōn*) “old” shows the same grade (cf. below).²

The verbal adjective from the root in the weak form (zero grade, with accent outside of the root or stem part) $*\acute{g}rh_2-nó-$ “ground” appears in OIA in *jūrṇá-* in the RV, and its younger form *jīrṇá-* “rotten, aged, frail”, Latin *grānum*, German *Korn*, Old-Church-Slavic *zrěno* “grain”. The phonological laws involved in these developments are: PIE $*\acute{g} >$ OIA j (OIran. z); $*\acute{r}h >$ OIA *ur/ir* before a vowel, *ūr/īr* before a consonant (OIran. $> ar$), Latin *rā*, Germanic *ur*, Slavic *r* (written *rb* or *rr*). Grassmann, of course, had a profound knowledge of comparative Indo-European linguistics; the best illustration of this is the famous Grassmann’s law about aspirate dissimilation in Indo-Iranian and Greek. But the existence of three laryngeals and the consequences they have left in Indo-European languages were not yet known to his generation. A strange reconstruction $*gvar$ was the result of the efforts of contemporary scholars who recognized the relationship between the words but could not find phonological regularities underlying their connection.

An autonomous adjective $*\acute{g}erh_2-ont-$ “old” is inflected in the singular: nominative $*\acute{g}erh_2-ōn(t) >$ gr. γέρων (*gérōn*) “old, old man”, accusative $*\acute{g}erh_2-ont-m >$ OIA *járantam* (cf. *Dictionary*, under participle), dative $*\acute{g}rh_2-nt-éi >$ OIA *juraté* (cf. *Dictionary*, under *jur*). *járatas* (genitive singular) and *járatos* (genitive dual) are results of analogical levelling after strong stem-form. The feminine stem is $*\acute{g}erh_2-nt-ih_2- >$ OIA *járatī-*, attested in the instrumental plural *járatibhis*. [Gotō 1987, 153 n. 238].

The causative *jar-áya-ti* goes back to PIE $*\acute{g}orh_2-éie-ti$, formed with the causative suffix $*-éie-$ added to the root in the $*-o$ -grade. It serves as the explicit substitute of $jár^i-a-ti$ in the meaning of ‘make old’, or as factitive to *jūr-ya-ti* ‘become old’. Short *a*

² “Laryngeals” are laryngeal consonants postulated in Proto-Indo-European; three sorts are assumed, h_1 (with non-colouring effect), h_2 (with *a*-colouring effect on the basic vowel $*e$), and h_3 (with *o*-colouring effect on $*e$). They correspond to a large extent to “coefficient sonantique” theoretically assumed by [de Saussure 1879], and have been made common property through articles by É. Benveniste, especially J. Kryłowicz 1927–1928 (cf. [Kryłowicz 1935; 2004]), and others. The new impulse for the theory has been given from the discovery of Hittite in 1915, where some remainders are found in the sound *h* (corresponding to h_2 in today’s theory). Cf. [Mayrhofer 1981].

in *jar* is due to Brugmann's law, i.e. PIE **o* becomes *ā* in Proto-Indo-Iranian in the open syllable, but to *a* in the closed syllable (syllable ending in a consonant). *jār-āya-ti*, with long *ā* is an analogic form after the type, for example *yātāyati* 'settle someone in his position' to *yātati*.

jūr-ya-ti in the RV with its younger variation *jīr-ya-ti* (Atharvaveda+) is a present stem characterized through the suffix **-je-* as a "fientive" formation, i.e. a stem denoting the change of the subject's state or condition as opposed to "doing something" (facientive), in this case, thus, 'become old, frail'.

jūr-va-ti 'wear, exhaust' is a **-uē-* present (PIE **ǵr̥h₂-uē-*) with a secondary accent. This present formation is rare and its accurate function is yet to be discovered. Grassmann properly explains the form as belonging to the group of *jar*ⁱ, *jur*, *jūr* (and *jīr*). Mayrhofer, who once judged this present stem differently [Mayrhofer 1956, 443], writes today [Mayrhofer EWAia, vol. I, 597], referring to [Gotō 1987, 153 n. 237], "correctly already Grassmann 499 ('to wear out'), who, however, arrives at the meaning 'to scorch, consume through the heat' because of repeated combination of *jūrv* with Agni [the fire god] (or the sun...); from there the false connection with *jvar*ⁱ (*jval*ⁱ) ['to scorch'] follows (Altindische Grammatik I 24...)." Grassmann owes his right judgement to the detailed treatment of all forms and occurrences as well as rational sorting out of the forms and meanings. He operates over the total figure of a word which was achieved in this way, towards its primary or central meaning. Thus, he could point out the history of a word group and possibilities of its development.

The interpretation of *ni-jūr-* played a decisive role in this case: RV II 29,6 *trādhvam no devā nijūro vṛkasya trādhvam kartād avapādo yajatrāḥ*. Grassmann translates [RV I, 38]: "Save us that a wolf does not devour us, that, ye reverends, we do not fall into a pit! [Errettet uns, dass nicht der Wolf uns fresse, dass, Hehre, nicht wir in die Grube fallen]"; my translation: Save us, ye gods, from the wearing down [the herd of sheep] by a wolf! Save [us] from falling down into a pit, ye revered ones, cf. [Gotō 1987, 153 n.237]! Grassmann's translation observes the syllables' number in the original and uses iambic and trochaic rhythm imitating the original metre to some extent. Here he shows his faithful approach to the original in a way which is different to the analytical approach he takes in his *Dictionary*.

It is in itself a quite natural process to control all the forms and usages, to analyse them, and construct them into a transparent complex. Grassmann could perform this procedure ideally and rationally. He represents these operations in one format, i.e. in a synchronic list of the forms and meanings. We can observe every step of the operation in his list, which helps us to go deeper into the history of words and forms.

2. Meaning entries

Among the meanings Grassmann gives under *jar* (column 479), (1) 'grind down, make fragile, make old', and (2) 'let become old, i.e. let reach an old age' are facientive (or agentive)-transitive, i.e. the subject does some action or influences someone/something. The meanings (3) 'become fragile, become old, be consumed' and (4) 'be ground down' are fientive-intransitive, i.e. a change of state or condition happens in the subject. (5) "causative" 'grind out, wear out, make old' is factitive: the subject makes someone/something do something or something happen. With "*jur* (1) become fragile, become old (of living beings)", and "(2) the same (of the god's deeds)" a fientive-intransitive meaning of the verb is meant.

The root-form *jur* (column 493) is observed from a facientive-transitive realization: "(3) make frail, make old, especially (4) in participle *jūrñá-* friable, brittle (of a net), frail (of the cast off snake's skin), (5) *jūrñá-* very old without nuance of fragility". For (4) and (5), also a fientive or essive (statutive) interpretation (the subject is situated in some state) may be possible. But the interpretation of Grassmann is correct, because the root starts from a facientive meaning. That is why a simple present stem *jár-a^{ti}* can mean 'make old', and a characterized stem *jūr-ya^{ti}* is formed specially for the fientive meaning 'become old'.

Morphological means are used to vary the lexical content of a verbal root into various meanings and realizations. There are also analogical or levelled secondary formations. Today we could represent the forms and meanings in a combination of morphological and noematical (i.e. relating to lexical components) entries. But such a process is in some parts still the subject of monographic studies. Perhaps we will have to wait until a dictionary is able to incorporate the results of the research (a scholarly dictionary, not a translation dictionary). Thus, currently one will have to hope that the user is well informed about morphology.

The meanings are listed and classified well, based on external (syntactical) and internal criteria. This makes it possible to verify primary and developed meanings or uses. Grassmann's interpretations have been in some ways and in some places improved or varied upon. Still, on the whole, we remain within the framework which he set. If scholars do not arrive at a common solution, they still resort to "according to Grassmann, so-and-so".

3. Form entries

Grassmann takes up indicative, "conjunctive", optative and imperative for the moods of the verb. His "Conjunktiv" means subjunctive and injunctive ("the second conjunctive", p.VI) today. However, it scarcely disturbs us. The forms and functions of

the injunctive were exactly ascertained only by Karl Hoffmann [Hoffmann 1967; first manuscript-version in 1950]. For a precise interpretation, whether a form is subjunctive or injunctive, an investigation is still necessary. Otherwise, the issue remains partly undeterminable. Thus, it was not a task of the dictionary at that time to make it clear whether a form is subjunctive or injunctive. The field of the stem formation and its ratio has made great progress up to the present day. Grassmann's judgement as aorist or present (including the imperfect, i.e. the preterit of the present stem which expresses a verb's action in its progressive aspect) cannot be accepted in all instances, especially in the case of the aorist stems which are used for the aspect without time-expansion. It is important, still, that the stems are sorted out separately. We can surely start our research from the *Dictionary*.

In frames and details of the nouns, pronouns, and adverbs, the views Grassmann presents are barely in need of revision today. Only some lemmas are to be put together, as is the case with the above-mentioned *járant-* and *jurat-*.³ To divide one lemma into several is much more difficult than to fulfil the procedure in the opposite direction.

There may be cases, in which some forms are to be distinguished. In the case of verbs, for example: The forms of the secondary root *dad*, *dáda-te* "hold" [Wackernagel 1942, 163f.] are registered under *dā* "give". We distinguish two homonymous roots ¹*yā* "travel, drive" and ²*yā* "beg" [Schmid 1956, 222ff.] among the forms Grassmann gives under "*yā* go, wander ... drive; arrive ...; approach begging ...; attack, pursue ...". Grassmann's '*cit* perceive, recognize', contains two separate roots ¹*cit* 'recognize, pay attention to' and ²*cit* 'shine, distinguish' [Gotō 1987, 137–141; Mayrhofer EWAia I, 547f., 548f.]. The preset stem *cáy-a-te* 'punish' under '¹*ci* place together, pile up' belongs to another root *ci*, which goes back to PIE **k^wei* 'receive sanction' together with Gr. τίνω 'pay, atone for', τίνομαι 'make atone', ποινή 'penitence' [Gotō 1987, 132; Mayrhofer EWAia I, 532f.]. *math* 'steal' (present *mathnāti*) and *manth* 'twirl (in order to make fire, etc.)' (*mánthati*) were distinguished by [Narten 1960, 121–135]. Grassmann's inventory of forms and meanings serves, naturally, always as basic material for such renovation.

We know that there are three separate roots among the forms under "*ar*, *ṛ* set in motion". They go back to PIE **h₁er* 'come into, reach at', **h₂ar* 'fit' and **h₃or* 'set oneself into a (quick) motion', cf. [Kümmel in Rix et al. 2001, 238, 269f., 299f.]. They are, however, not yet duly distinguished, so every time we go back to Grassmann.

³ Belong or go back to one and the same paradigm, e.g.: *rāyi-* 1149f. and *rāi-*, *rā-* 1182ff. 'wealth'; *dāru-* 595f. and *drú-* 647f. 'wood'; *sānu-* 1511 and *snú-* 1606 'back'; *bṛid-* 1678 and *bārdi-* 1661 'heart'; *āsan-* 137 and *aśman-* 139 'rock, stone'; *dvār-* 651 and *dūr-* 613 'door'; *kṣām-* 363, *kṣā* 365, *kṣmā* 371, *kṣāman-* 365, *gmā-* 415 and *jmā-* 502 'the earth'. Under *star-* 'star' one misses *tāras* 'stars' (nominative plural) VIII 55,2. *dyāv-*, *dyu-* and *div-* are duly registered together under *div-* 'heaven' (601ff.).

The treatment of nouns for 'great, big' *máh-*, *mahá-*, *mahánt-* (Grassmann "*mahát*"), and *mahā-* is very impressive. The occurrences are very numerous and the interpretation is difficult. Indeed, we may meet with cases which should be better understood in another way, but still Grassmann's data remain essential for such precise study. *mahnā*, *mahānā*, and *mahābbis*, for which the *Dictionary* gives a lemma "*mahán-* neuter (from *mah-*) greatness, fullness, power, magnificence", are interpreted today as instrumentals of the masculine *mahimán-*.⁴

Metrical analysis

Grassmann makes efforts throughout to give, beside the forms transmitted in the text, also the forms which are to be assumed for metrical reasons in the original poetry. On the whole, we can rely on his assumptions. But each case has its own difficulty. Among others, Oldenberg has performed philological examinations in this field [Oldenberg 1888, 1909, 1912].

The striving of the generations following Grassmann and Oldenberg consisted of explaining such phenomena linguistically: exacter versions of the rules of syllabic structure,⁵ identification of different consequences of disappeared sounds (three laryngeals), also in morphological elements.⁶ The poets, in addition, had their "poetic freedom, poetic licence" to use older and newer forms of different times. *jána-*, for example, is a common word for "man, people". The stem is scanned as short-short or long-short. The latter is explained from the older constellation **jóh₂|no-* consisting of a closed (thus long) and a short syllable.⁷

⁴ Cf. [Mayrhofer 1986, 159; Schindler 1977, 57; Tichy 1986, 99 n.7; Gotō 2005, 209]. The appearance and disappearance of *i* in *mahimán-* and *mahmn-* is explained from the "loss of a laryngeal" today. *mahām* (column 1016b under *mahát-*) belongs to "*mahā* adjective 'great' (from *mah-*)" (column 1018).

⁵ Sievers' law, now enriched by a more precise version of Schindler's syllabification law [Schindler 1977, 56]; some individual laws such as those of [Stang 1965, 292ff.; Lindeman 1965, 38ff., 69ff.]. Cf. [Mayrhofer 1986].

⁶ E.g. nominal suffix **-b₃on-*/*-b₃n-* (Hoffmann's possessive-suffix), **-ih₂o-*, etc., the optative suffix **-ieh₁-*/*-ih₁-*/*-ih₁-*. The OIA roots are divided traditionally into two groups *seṭ* und *aniṭ*. They are interpreted today, respectively, as the roots ending with a laryngeal and otherwise.

⁷ *svār-* 'sun's light' is delivered in the tradition only in this form. Grassmann registers it as "(*svār*), *súar*, *sūr*", it means: attested as *svār-*, but to be read as *súar* (better *súvār*, we write *súvār* with a subscript character as the sign to be supplied). The word is reconstructed as **-l/n-* heteroclite neuter of substance: nominative (A) **sáh₂-u_l* (> Proto-Celtic **sāual* > **hōual* > **hōu_l* > Middle-Breton *heol*), changed into (B) **sáh₂-u_{el}* after other **-l/n-* or **-r/n-* heteroclites, or into (C) **sh₂u_uel* (> OIA *svār*-, *súvar*-, OAv. *huu_{ar}*²) after genitive **sh₂-u_{en}-s* with Lindeman's variant *sh₂-u_{en}-s* (> Old-Avestan **huu_{en}ang*^b > x^o *ang*, Young-Avestan *hū*). OIA *súvār* represents a nominative form (C). Only in RV II 35,6 the word *svār* is scanned really as

Prepositions, particles, etc.

An important advantage of the *Dictionary* is its careful and thought-out treatment, as well as its rational presentation of modal words such as prepositions and particles which are decisive for understanding the “logic” of the poets. In order to illustrate how Grassmann treats such small words, let us cite the item about *ā* ‘towards something/someone’, used when the directional sense is “towards here”, i.e. seen from the terminal:

column168–171 *ā*:

... The basic notion is “on”, related first of all to a limit or surface, to the immediate proximity. Therefore it conveys the meaning of the adverb of the direction “up this way, near to; hither” (i.e. moving to the immediate proximity of the speaker), or “upwards, up to; towards” (moving to the immediate proximity of the imagined object). Therefore it functions as a preposition with a following ablative “up to, as far as”, while it only emphasizes the meaning of the cases if it is used as a preposition with preceding accusative, ablative, locative. However, this word order is not unchangeably fixed in both cases. It connects words as a connecting particle in the meaning of “and”, i.e. “attaching oneself to the expressed one arriving at this, stepping hereto”. Finally, it serves to emphasize the preceding accented word in the sense that its notion is valid in its full contents, in its just proper meaning. Thus it is realized as:

Word of direction with the verbs: xxx xxx xxx xxx xxxxxx xxx xxx. [“xxx” suggests omission by the article’s author, as above in 1; here, the verbal roots are listed to which *ā* is added.]

- Prepositions with preceding accusative: (1) ‘up to’ (of space) xxx xxx xxx xxx [the occurrences are omitted by the author]; (2) with an abstract noun: ‘towards, aiming at (expressing the goal)’ xxx xxx xxx; (3) connected with *ānu* ‘along, after’, see under *ānu*.
- Preposition with following ablative: (1) ‘up to, as far as’ (of space) xxx; (2) (of time) xxx xxx; (3) ‘for, for the sake of’ xx; (4) ‘hither from ...’ xx.
- Preposition with preceding ablative: (1) ‘hither from ...’ (of space) xxx xxx xxx; (2) ‘from’ (of space) xxx xxx; (3) ‘originated from ...’ xxx; (4) ‘(to make) of ...’

monosyllabic, and used as a genitive. There is an echo of the lost genitive form **sh₂-uén-s* behind the tradition (cf. Gotō in [Witzel and Gotō 1987, 814]). The form “*sūr*” Grassmann cites is related, e.g., to the genitive *sūr-as*, and besides this dative *sūr-é* with legitimate accent. They are mixed formations of *suur-* as an artificial weak form of *súvar*, and a normal genitive ending *-ás*.

xxx; (5) 'from (the heart)' xxx; (6) 'away from ...' xxx; (7) 'before, above (others)', i.e. 'in higher, superior grade to others' xxx xxx; (8) 'towards ...' against the expected word-order xxx; (9) *ádhi á* 'from ... hither' see under *ádhi*.

- Preposition with preceding locative: (1) denoting the place in which something exists, or at which something reaches (in case of verbs of motion), thus 'on, at, in, by, to' xxx xxx xxx xxx xxx; (2) 'in the case of' (some action), or 'in' (some state) xxx xxx; (3) of time: 'on, in' xxx; (4) *antár á* ['mid in ...'], *sácā á* ['together with ...'], see under *antár* and *sácā*.
- Preposition with adverbs that stand for case-forms: xxx. Combinations with other prepositions, see under the cases.
- Conjunction 'and' (1) between two jointed words: xxx xxx; (2) after the last element in case of multiple enumeration: xxx; (3) after the word (or words) in the jointed part which is identical with that in the first part according to wording or meaning, for example xxx xxx; (4) at the end or before the closing verb of the jointed part: xxx xxx.
- Emphasizing particle (1) after words of number or grade in order to express that the number or grade is reached completely: xxx; especially in the formula *trís á divas* 'three times each a day' xxx xxx; (2) thus after adjective or adverb which can be emphasized in order to express a really high grade, 'really': xxx xxx; (3) beside substantive and adjective in the sense of 'representing oneself really as such', 'really as, really like': xxx xxx; after verbs: xxx xxx.

For elaborating categories such as 'preposition, particle, adverb, conjunction', further considerations and precisions might be necessary. For a dictionary, still, Grassmann's arrangement is absolutely substantial: preposition (governing the case), conjunction (small words for junction of words, sentences, or parts of a sentence, etc.), and particle (small words which express the speaker's attitude to the whole contents of a sentence), in addition, preverb and adverb. His verification and representation of the internal (meaning) and external criteria (case government, word order) are rational and compact. Here we have what remains as probably one of the best descriptions in this field even today. The works of Delbrück (1842–1922) are an irreplaceable basis in descriptive and comparative syntax, i.e. the philological, comparative and historical syntax in Indo-European languages. Delbrück [1888] often summarizes Grassmann's descriptions about prepositions in the RV, saying "I have nothing substantial to add to what Grassmann writes", and describes the findings from the prose literature of the Veda. Delbrück tries to treat the prepositions in connection with verbs as much as possible. In this sense, the basic meaning emerges more clearly in Grassmann's *Dictionary*.

One weaves the logical parts of a sentence with the help of prepositions and connecting particles (conjunctions). With the help of the modal particles the speaker adds

his own attitude to the contents expressed in his speech, or “wraps” his sentence’s substance. The same is applicable to the modal categories of the verbs. Grassmann is engaged in this domain with special intensity and shows his strength for providing rational and compact layout. This is due to his efforts to understand and reproduce an old foreign literature. It is very impressive that we feel as if there were nothing foreign in Grassmann’s interpretation. It is as if Grassmann tried to understand the thinking and expressions of old Indian folk as his own language. He wanted to understand the RV in its entirety. For this, he did not use a poetic intuition or veil, but analyzed the expressions into their elements on various levels and constructed the thus-gained results in a clear structure. Grammar, which is not contained in school books, but in our brains, plays the decisive role. It reminds me of the method of Aristotle, but in the modern world. He has, in short, utilized all the methods of philology in an ideal way. Grassmann was conscious of the importance of providing accurate translations first, before attempting expositions of the contents and its background. His *Dictionary* is still an exemplary work on a closed text. His arrangement and the whole edifice of the work have a universal value.⁸

Abstract language and German

I do not know to what extent our logic itself depends on our mother tongue, and in some cases, also on other languages one can control if necessary. What Grassmann uses in his analysis seems to be an abstract language, as if, for example, a stranger would formulate something precisely with the most extreme intensity, but at the same time, completely. He does not avoid problems with German expression. His *Dictionary* is his message to the following generations showing how our brain should struggle and understand an appearance, an object in the aggregate. I am completely persuaded that Grassmann thought the human brain to be equipped commonly and universally since the time of the Rigveda, at the latest.

However, I must immediately add that the enterprise was made possible by German in a direct way. The fact that German preserves an old structure of the Indo-European languages and its word formation (root-suffix-ending, composition, preverbs

⁸ Bartholomae was one of the best Indo-European linguists of all time and one of the best specialists in the field of Iranian studies. His *Dictionary of Old-Iranian* [Bartholomae 1904] registers the form under each meaning with its occurrence. Grassmann’s principle, first the meaning with numerals, then the occurrences cited in the frame of forms with the figure of the meaning, is far more expedient. Bartholomae decided on his system probably because of the relatively small corpus of Old-Iranian texts. The inventory of the forms is important and makes things clearer. It is a desideratum to remodel the dictionary of Bartholomae after Grassmann’s principle.

and prepositions, which can also be laid on each other), the genus of nouns, relatively free word order, and the usage of modal verbs – all of this favours the literal reproduction of the original text of the RV. In extreme cases, it is possible to transport the wording of the Rigveda into German and only then try to understand it. This possibility favoured the *Dictionary*. The philology of the RV is still today strongly impressed by the German language and its way of logical representation. Also, Grassmann played an important role here, alongside the *Petersburger Dictionary* by Böhlingk and Roth (1855–1875), which was also a basis for Grassmann. With this reservation, one can say that the *Dictionary* is a most excellent exemplar in the analysis and representation of a closed corpus. It remains for us an active and irreplaceable instrument.

The decisive year of 1875

I would like to add some closing comments on the epoch we have been dealing with. I have not checked, nor am I informed about the question how much Grassmann owed to the *Petersburger Dictionary* [PW] (cf. contribution of Ms. Kozińska in this volume). The 7th and last volume (*ś, ś, s, h*) of the PW was printed on 7 August 1875, and has a preface dated 4 August 1875; the preceding volume (*y, r, l, v*) was printed on 8 December 1871. Grassmann's *Dictionary* has a preface dated 18 August 1872 and afterword, 8 August 1875. One may, however, safely assume from the friendship between Roth and Grassmann, which is attested by the afterword of the *Dictionary* and the letters [Petsche 2009, 167–185], that Grassmann was substantially informed of the whole material which the *Petersburger Dictionary* includes for the RV. Grassmann consulted the PW thoroughly, but he decided always for himself, as he himself says in the preface (p.V). He trusted only in his own reasoning. He did not rely on the Padapāṭha ('reading in words', i.e. segmentation into words of the RV-text, which is transmitted in contentious reading for recitation not separated word by word), by Śākalya, maybe about the sixth century B.C., nor the commentary of Sāyaṇa (fourteenth century A.D.). Grassmann profited, it is safe to assume, extensively from these traditional Indian contributions, but judged always only for himself.

The year of 1875, in which the PW and the *Dictionary* were completed, is thus an important date for the history of Indology. The next year, 1876, in which the 1st volume of Grassmann's translation of the Rigveda was published, is a revolutionary year which opened the way for the modern comparative Indo-European grammar with the monumental works of Leskien (the sound-change operates mechanically), Osthoff (the existence of PIE sonant **r*), Brugmann (the existence of PIE nasal sonants **ṇ*,

**m*, and Brugmann's law, cf. 1.1), Verner (Verner's law in Germanic), Sievers (foundation of Phonology), Winteler (the description of Swiss German dialect), etc. (cf. "The *annus mirabilis* 1876 and posterity" by [Hoenigswald 1978], and [Schmitt 2009, 15ff.]). Grassmann's efforts in philology and linguistics seem to have been somewhat independent of the very active scholarly mainstream. How would it have been if Grassmann had written a grammar? But he lived perhaps a little too early to take on this task with his customary precision.

Freguglia, Paolo; Prof. Dr.

Educational background: Study of mathematics and physics (Laurea in Matematica (1971), Sapienza University of Rome); Study of philosophy at Pontificia Universitas Gregoriana of Roma.

Current position: Full Professor, Dept. of Pure and Applied Mathematics, University of L'Aquila, Italy.

Main research interests: History of mathematics (algebra and geometry in the XVI and XVII century and in the XIX and XX century [geometric calculus]); Logic and foundations (modern aspects of syllogistics, inductive statistics, analysis of Peano's school contributions to foundations of mathematics); Mathematical models in physics (geometrical optics and beam dynamics) and in biology (geometrical and dynamics models in evolutionary theories).

Selected publications: Dall'eredità grassmanniana alla teoria delle omografie nella scuola di Peano. *La Matematica nella società e nella cultura. Journal of Unione Matematica Italiana*. 1 (2008), with C. Bocci. // Calcolo geometrico e numeri ipercomplessi: origini e primi sviluppi ottocenteschi. *Bollettino dell'unione matematica italiana*. A, 8, 7–A (2004). // *Geometria e numeri. Storia, teoria elementare ed applicazioni del calcolo geometrico* (book, 2006), with a contribution by A. Bazzani.

Address: Via Panoramica, 233, I – 58019 Porto S.Stefano GR, Italy.

E-mail: paolo.freguglia@technet.it or fregugli@univaq.it.

Gotō, Toshifumi; Prof. Dr.

Educational background: Study of Indology, Indo-European linguistics, Indo-Iranian philology, Dr.phil. in Indo-European linguistics (Erlangen).

Current position: Professor in Indology and History of Indian Buddhism at Tōhoku University, Sendai, Japan.

Main research interests: Indo-European studies, the extension of Indo-European speaking people; historical grammar of Old Indo-Iranian, especially in the verb; religion and Weltanschauung in the Veda, Avesta, and Buddhism.

Selected publications: Die "I. Präsensklasse" im Vedischen (1987). Vasiṣṭha und Varuṇa in RV VII 88 (2000). // "Purūravas und Urvaśi" aus dem neuentdeckten Vādhūla-Anvākhyāna (Ed. Ikari) (2000).

Address: Department of Indology and History of Indian Buddhism, Graduate School of Arts and Letters, Tōhoku University, Kawauchi 27–1, Aobaku, Sendai, 980–8576, Japan.

E-mail: gotop@sal.tohoku.ac.jp.